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V.I.Balbekov, S.V.Ivanov

THRESHOLDS OF LONGITUDINAL INSTABILITY OF BUNCHED BEAM IN THE PRESENCE OF DOMINANT INDUCTIVE IMPEDANCE

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Abstract

Balbekov V.I., Ivanov S.V. Thresholds of Longitudinal Instability of Bunched Beam in the Presence of Dominant Inductive Impedance: IHEP Preprint 91-14. - Protvino, 1991. - p. 24, fig. 5, refs.: 9.

At high intensity of the beam in a synchrotron a purely imaginary inductive impedance is, at worse, capable of maintaining sustained coherent oscillations of a bunch. But these can turn unstable given the presence of any additional impedance with a positive real part. As a matter of fact, such an impedance can always be found in the beam environment.

The paper studies thresholds of multipole instability of a bunch under the assumption that all its oscillational eigen-modes are completely determined by a dominating low-frequency inductance of the vacuum chamber. The acceptable value of its impedance (divided by the mode number) is found to coincide, though formally, with the well-known (Boussard) local criterion for stability of microwave oscillations multiplied by the value of relative inside-the-bunch spread of synchrotron frequencies. The effect of stationary space charge self-fields is also estimated.

Аннотация

Балбеков В.И., Иванов С.В. Пороги продольной неустойчивости сгруппированного пучка при наличии доминирующего индуктивного импеданса: Препринт ИФВЭ 91-14.- Протвино, 1991. - 24 с., рис. 5, библиогр.: 9.

При достаточно высокой интенсивности пучка в синхротроне чисто мнимый индуктивный импеданс может поддерживать только ненарастающие когерентные колебания сгустка. Однако эти колебания становятся неустойчивыми при наличии любого дополнительного импеданса с положительной вещественной частью. Фактически, такой импеданс всегда присутствует в окружении пучка.

В работе исследуются пороги мультипольной неустойчивости сгустка в предположении, что все его собственные моды колебаний полностью определяются доминирующей низкочастотной индуктивностью вакуумной камеры. Показано, что допустимое значение её приведённого импеданса формально совпадает с известным локальным критерием устойчивости микроволновых колебаний, умноженным на относительный разброс синхротронных частот в пределах сгустка. Оценено влияние стационарных эффектов пространственного заряда.

A purely imaginary inductive impedance takes into account local E.M. fields carried by a beam. Such an impedance is unable to cause the beam instability in its "proper" sense, i.e. exponentially growing oscillations of bunches. At a sufficiently high beam intensity in a synchrotron this impedance is, at worse, capable of maintaining sustained coherent oscillations of a bunch. But this motion turns unstable in the presence of any retarding fields (e.g., of resonant cavity fields). Formally, the latter fields are treated in terms of an impedance with a positive real part.

The paper proceeds from the assumption that it is precisely the local type of interaction which dominates in the self-consistent beam dynamics. Thus, the effect of retarding fields is reduced to a small perturbation which, nevertheless, shifts the real frequency spectrum of the first-order approximation to an unstable domain. Within the frames of such an approach, the explicit nature of retarding fields becomes inessential.

The similar problem for the transverse motion was studied earlier in ref. /1/.

1. INTRODUCTIVE REMARKS

Let ϑ be an azimuth in the co-moving coordinate system. (Namely, $\vartheta = \theta - \omega_s t$ where θ is a generalized azimuth along the ring, ω_s is the angular velocity of a synchronous particle, t is the time.) Let the origin $\vartheta = 0$ of the ϑ-coordinate frame be placed on the synchronous particle of the bunch in question. Consider the pair $(\vartheta;\vartheta' \equiv d\vartheta/dt)$ as canonical variables. The single-particle Hamiltonian is taken to be

$$H(\vartheta,\vartheta') = \Omega_0^2\, U(\vartheta) + \vartheta'^2/2, \tag{1}$$

where $U(\vartheta)$ is the potential energy (or well) of longitudinal motion, for definiteness $U(0) = 0$; Ω_0 is the typical value of the synchrotron frequency (angular). If $U(\vartheta) \simeq \vartheta^2/2$ when $\vartheta \to 0$, then the value of Ω_0 coincides with the synchrotron frequency for small amplitudes. All the parameters of an equilibrium bunch are supposed to be evaluated with the effect of stationary (incoherent) space charge self-fields taken into account.

Let us describe the distribution of particles inside a bunch by a function $F(\mathcal{E})$ where $\mathcal{E}$ is the energy of longitudinal oscillations which coincides numerically with Hamiltonian (1). Let us impose the following normalization on $F(\mathcal{E})$:

$$\int_0^\infty F(\mathcal{E})\,\frac{d\mathcal{E}}{\Omega_s(\mathcal{E})} = \frac{\mathcal{E}_0}{\Omega_0} \tag{2}$$

where $\mathcal{E}_0$ is the energy of oscillations on the bunch boundary.

The commonly accepted approach to beam instabilities[2] is the study of an infinite system of equations in terms of amplitudes $J_k(\Omega)$ of the beam current perturbation harmonics $J_k(\Omega)e^{ik\vartheta - i\Omega t}$:

$$J_k(\Omega) = R^{-1} \sum_{l'=-\infty}^{\infty} Y_{kk'}(\Omega)\left[Z_{k'}(\Omega)/k'\right] J_{k'}(\Omega), \tag{3}$$

$$k = n+lM, \quad k' = n+l'M, \quad -\infty < l,l' < \infty, \quad k,k' \neq 0.$$

Here $n = 0,\ldots,M-1$ are the indices of the collective modes of beam oscillations which are discriminated by the value of bunch-to-bunch phase shift of coherent oscillations $(\Delta\psi = 2\pi n/M)$, M is the number of identical and equally spaced bunches in the beam, Ω is the coherent frequency in the co-rotating system, $Z_k(\Omega)$ is the vacuum chamber impedance.

Eq. (3) embeds the numerical factor R dimensioned as an electric resistance:

$$R = \frac{\beta^2\eta\,(m_0c^2\gamma)}{e\,J_0}\left[\frac{\Delta p}{p_s}\right]^2 \tag{4}$$

where β and γ are the Lorentz factors, $\eta = \alpha - \gamma^{-2}$, α is the orbit compaction factor, $(m_0c^2\gamma)$ is the total energy of a particle, J_0 is the beam current averaged along the orbit, $\pm\Delta p/p_s$ is the fractional momentum spread on the phase-plane trajectory $H = \mathcal{E}_0$ of the bunch boundary.

The so-called dispersion integrals Y can be written down as a series in multipole excitations of the bunch:

$$Y_{kk'}(\Omega) = \frac{i}{\pi}\sum_{m=-\infty}^{\infty}\int_0^\infty \frac{m\Omega_0}{\Omega - m\Omega_s(\mathcal{E})}\left[-F'(\mathcal{E})\right] I_{mk}(\mathcal{E})\, I^{*}_{mk'}(\mathcal{E})\,d\mathcal{E}. \tag{5}$$

Here $\Omega_s(\mathcal{E})$ is the frequency of incoherent synchrotron oscillations along the phase-plane trajectory $H = \mathcal{E}$. The quantities

$$I_{mk}(\mathcal{E}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{im\psi - ik\vartheta(\mathcal{E},\psi)}\,d\psi, \tag{6}$$

where ψ is the phase of synchrotron oscillations, can be referred to as the coefficients of a series expansion of a plane wave in the multipole excitations.

2. INTEGRAL EQUATION

The problem under study (namely, the wide-band impedance) can be treated in a more convenient formulation – in terms of Lebedev's integral equation[3]. The latter can be constructed by applying the inverse Fourier transform in ϑ-variable to Eq. (3):

$$J_n(\vartheta,\Omega) = \sum_{l=-\infty}^{\infty} J_k(\Omega)\, e^{ik\vartheta}, \quad k = n+lM. \tag{7}$$

As a result, one arrives at the following integral equation:

$$J_n(\vartheta,\Omega) = \frac{i}{M\pi R}\int d\vartheta_1\, K(\vartheta,\vartheta_1,\Omega)\int d\vartheta_2\, W_n(\vartheta_1 - \vartheta_2,\Omega)\, J_n(\vartheta_2,\Omega), \tag{8a}$$

$$K(\vartheta_1,\vartheta_2,\Omega) = \sum_{m=-\infty}^{\infty} \int \frac{m\Omega_0}{\Omega - m\Omega_s(\mathcal{E})} \left[-F'(\mathcal{E})\right] \frac{\Omega_s^2(\mathcal{E})}{\pi^2} \frac{\cos m\psi(\mathcal{E},\vartheta_1)\cos m\psi(\mathcal{E},\vartheta_2)}{|\vartheta'(\mathcal{E},\vartheta_1)|\;|\vartheta'(\mathcal{E},\vartheta_2)|} d\mathcal{E} \tag{8b}$$

Integration over $d\vartheta_{1,2}$ is performed within the boundaries of the single bunch under study. In the kernel K, this integration is carried out over the region $\mathcal{E} > \max(U(\vartheta_{1,2}))$ where both functions $\vartheta'(\mathcal{E},\vartheta_{1,2})$ and $\psi(\mathcal{E},\vartheta_{1,2})$ do exist. For $Im\Omega \to +0$ the residual is got around by means of the standard Landau's prescription. The quantity W_n is the equation symbol for the inverse Fourier transform of the reduced impedance $Z_k(\Omega)/k$, this transform being performed over the azimuthal harmonics of the n-th mode only:

$$W_n(\vartheta,\Omega) = M \sum_{l=-\infty}^{\infty} \left[Z_k(\Omega)/k\right] e^{+ik\vartheta}, \qquad k = n+lM. \tag{8c}$$

From now on, we proceed from the following crucial assumption. Let us suppose that the beam environment is dominated by the impedance

$$Z_k(\Omega)/k = i\omega_s L \;, \tag{9}$$

where L is the effective "inductance" of the vacuum chamber. This inductance is usually presented as a sum of two terms, $L = L_1 + L_2$, see[4,5] and some references therein. The first term takes into account the effect of a perfectly conducting smooth chamber

$$\omega_s L_1 = \frac{Z_0 g_0}{2\beta\gamma^2} > 0 \tag{10}$$

where $Z_0 = 120\pi$ Ohm is the free-space impedance, $g_0 \sim 1 \div 10$ is a geometrical factor. This component brings about a small contribution when $\gamma >> 1$. Another term appears to be more essential. It is a low-frequency inductance L_2 caused by the vacuum chamber cross-section irregularities. It can be evaluated experimentally, e.g. by means of potential-well bunch lengthening or shortening effect. In the existing accelerators with the usual (non-smoothed) vacuum chambers $\omega_s L_2 \simeq -(10 \div 30)$ Ohm[5,6]. The net inductance L being the sum of two components, its value can possess an arbitrary sign.

Of course, the introduction of the very concept of inductance L_2 is justifiable for some band of low frequencies, the relevant restrictions are touched upon at the end of Sect.3 of the paper.

On substituting Eq.(9) into Eq.(8c) one gets

$$W_n(\vartheta,\Omega) = 2\pi i\;\omega_s L\; e^{in\vartheta}\;\delta_{2\pi/M}(\vartheta) \tag{11}$$

where $\delta_{2\pi/M}(\vartheta)$ is a periodic sequence (with $2\pi/M$ as a period) of delta-functions. As a result, Eq.(8a) reduces to the integral equation with a symmetric kernel:

$$\left[\lambda(\Omega)\right] \times J(\vartheta) = -\,2\,\frac{\omega_s L}{M\,R} \int d\vartheta_1\, K(\vartheta,\vartheta_1,\Omega)\, J(\vartheta_1). \tag{12}$$

As it could be expected, here the dependence on n-index disappears. It is by virtue of the locality of interaction (11) that the relative motion of bunches becomes inessential.

Eq.(12) was first studied in ref.[3], though synchrotron frequency spread not being taken into account.

The dispersion equation for coherent oscillations can be written down in the form of equality:

$$1 = \lambda_i(\Omega) \tag{13}$$

where $\lambda_i(\Omega)$ is an eigen-value of Eq.(12). On solving Eq.(12) in terms of Ω, one finds the coherent frequency for the relevant eigen-mode $J_i(\vartheta,\Omega)$ of bunch oscillations. The existence of such solutions with a positive imaginary part of Ω is an evidence for the beam instability.

We shall study Eq.(13) by a generalized threshold map technique. This approach is based on a graphic analysis of a family of closed threshold curves $C_i(\Omega)$ which can be interpreted as images of the straight line $\left[Im\Omega \to +0\;;\; -\infty < Re\Omega < \infty\right]$ subjected to multiple-valued transform $\lambda_i(\Omega)$. The whole system is stable under the condition that all these curves do not cross the interval $(1,\infty)$ belonging to the real axis of the complex plane λ. In other words, the point $1+i0$ should not occur within a region encircled by any branch of the threshold curve $C_i(\Omega)$.

Consider some general features of these threshold curves. We begin by writing down an eigen-value $\lambda_i(\Omega)$ as a quadric functional in terms of the relevant eigen-function $J_i(\vartheta,\Omega)$:

$$\lambda_{\ell}(\Omega) = 2\,\frac{\omega_s L}{M\,R}\sum_{m=-\infty}^{\infty}\int_0^{\infty}\frac{m\Omega_0}{\Omega - m\Omega_s(\mathcal{E})}\Big[-F'(\mathcal{E})\Big]\,|R_m(\mathcal{E}|J_{\ell})|^2 d\mathcal{E}, \qquad (14a)$$

$$R_m(\mathcal{E}|J) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\cos m\psi\; J\Big[\vartheta(\mathcal{E},\psi)\Big]d\psi\,, \qquad (14b)$$

$$\int |J_{\ell}(\vartheta)|^2 d\vartheta = 1. \qquad (14c)$$

Let us restrict ourselves to single-peaked distributions $F(\mathcal{E})$, $F'(\mathcal{E}) < 0$ and monotonous functions $\Omega_s(\mathcal{E})$. First, consider the mapping of the following segments of the real axis of the complex plane Ω:

$$\Omega_{s\ min} < \frac{Re\Omega}{m'} < \Omega_{s\ max}\,, \quad Im\Omega \to +0 \qquad (15)$$

where $(\Omega_{s\ min} \div \Omega_{s\ max})$ is inside-the-bunch band of incoherent synchrotron frequencies, m' is an integer. One can easily see that m' determines the number $m = m'$ for the term of series (14a) which contains the resonant denominator running through the zero value. Formally, it presents an evidence for the strong (resonant) excitation of the bunch oscillations at the m-th multipole on frequencies Ω (15). We shall treat the relevant singularities in the kernel of Eq.(14a) in a usual way, i.e. the principal value integral plus the residue in the pole. Physically, it can be interpreted as an account of Landau damping in the bunch. It can be easily seen that, under assumptions (15), the curves $C_{\ell}(\Omega)$ are entirely running either in the upper, or in the lower half-planes of the complex plane λ:

$$Im\lambda_{\ell} \neq 0, \qquad \mathrm{sign}(Im\lambda_{\ell}) = \mathrm{sign}(R\,L\,Re\Omega). \qquad (16)$$

Whereof:

(a) Eq.(13) has certainly no solutions at intervals (15). From a physical viewpoint, this means that the inductive impedance under study can never excite resonantly the particles which are rotating along the inner phase-plane trajectories of the bunch.

(b) The curves $C_{\ell}(\Omega)$ do not cross the real axis, the only exception being for the mapping of some close vicinity of the point $Re\Omega = 0$. The discussion of such a peculiarity of the threshold curves is closely related to the the problem of

attainability of the Bunched-Beam Negative-Mass Instability Threshold in view of the simultaneous effect of stationary space charge self-field introduced by the same impedance[3]. We shall not discuss this question here, the problem being essential for much higher beam intensities as compared to the ones treated in the paper.

(c) On Eq.(15) being violated, the eigen-values $\lambda_{\ell}(\Omega)$ gain purely real values. Thus, at sufficiently high beam intensity Eq.(13) can provide only sustained solutions with real coherent frequencies lying beyond intervals (15). To achieve these solutions, one should go beyond some threshold intensity value, this threshold being introduced by Landau damping.

One can easily imagine the general appearance of the threshold curves $C_{\ell}(\Omega)$ (see Fig.1). They are made up of a sequence of closed loops which run along the axis $Im\lambda = 0$ near the origin, and are closed up in the half-planes (16). The whole picture is symmetric

Generalized threshold map

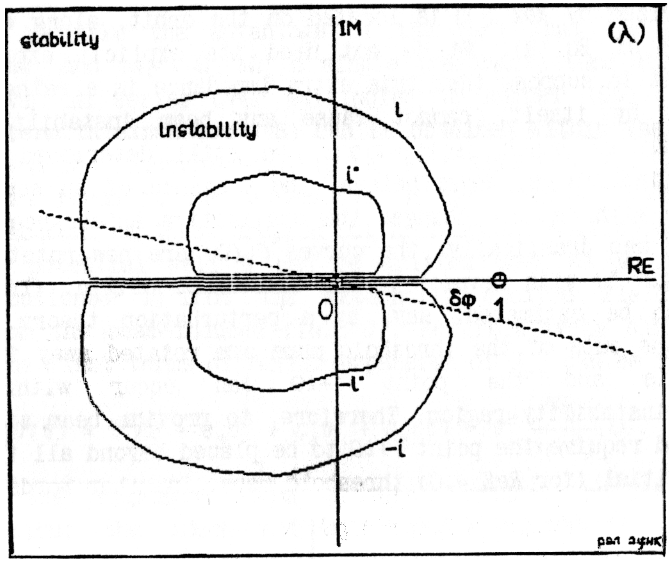

Fig.1. A sketch of threshold curves.

with respect to the real axis. Each loop corresponds to the excitation of a certain eigen-mode of the bunch oscillations. Unstable parameters (i.e. providing $Im\Omega > 0$) are located inside the regions encircled by separate loops. These could have represented physically feasible oscillations, should the r.h.s. of Eq.(13) have been a complex quantity. Each straight-line segment of the axis $Im\lambda = 0$ containing the real portion of curve $C_i(\Omega)$ can be interpreted as a cut on the threshold map for mode No."i". Indeed, any point of this cut is juxtaposed to a pair of solutions of the dispersion equation with $Re\Omega_i = - Re\Omega_{-i} > 0$.

At sufficiently high beam intensity the point $1+i0$ can occur within the cuts of the threshold map. This shows evidence for a possibility of non-growing (as $t \to \infty$) oscillations with a real coherent shift. That is, a purely imaginary impedance can never cause beam instability in its "proper" sense. Nevertheless, this situation in no way means that an arbitrarily high level of the low-frequency inductance of the vacuum chamber is practically tolerable.

To prove this, let us suppose that some additional impedance characterized by $ReZ > 0$ is located on the orbit, along with the impedance of Eq.(9). We do not need its explicit expression. Suffice it to suppose that this extra impedance is a rather small one and, by itself, cannot cause any beam instability. The eigen-modes of bunch oscillations are still determined by the dominant inductance. Nevertheless, the presence of an additional impedance with $ReZ > 0$ changes the qualitative appearance of the threshold map drastically: the curves $C_i(\Omega)$ are now rotated at a small angle $\delta\varphi$ with respect to the coordinate origin. (The value of $\delta\varphi$ can be estimated, say, by a perturbation theory.) As a result, the cuts of the threshold maps are rotated away from the real axis and the point $1+i0$ can occur within the "proper"-instability region. Therefore, to provide beam stability one should require the point $1+i0$ to be placed beyond all the cuts of the initial (for $ReZ = 0$) threshold maps. In other words,

$$\max_i Re\lambda_i(Re\Omega + i0)\Big|_{Im\lambda_i=0} < 1. \qquad (17)$$

Thus, the first-order estimates of the unknown threshold parameters require, formally, the evaluation of coordinates of the

extreme points of the cuts. Rigorously speaking, the curves $C_i(\Omega)$ are asymmetric with respect to the imaginary axis. Therefore, the instability thresholds would differ for the accelerator operation regimes either below, or above transition. (At transition, $\lambda_i(\Omega)$ reverses its sign.)

3. *BUNCHES WITH SMALL NONLINEARITY*

It is this case which presents a major practical interest. Let $\Delta\Omega_s = \Omega_s(0) - \Omega_s(\mathcal{E}_0)$ denote the total inside-the-bunch spread in synchrotron frequencies. The quantity $\Delta\Omega_s$ can have any sign. For definiteness, we take $\Delta\Omega_s > 0$ which corresponds to the natural nonlinearity. We shall construct the solution to the eigen-value problem (12) which is asymptotically correct for $\Delta\Omega_s/\Omega_0 \to 0$. We are looking for the first two terms in the series:

$$\lambda = a_{-1}\left(\frac{\Delta\Omega_s}{\Omega_0}\right)^{-1} + a_0 + \dots . \qquad (18)$$

Let us take the advantage of the fact that for $\Delta\Omega_s/\Omega_0 \to 0$, $Re\Omega \simeq m\Omega_s$ and $|m| < \Omega_0/\Delta\Omega_s$ a single (resonant) term indexed by m dominates in series over multipoles in kernel (8b). Thus, the first term in expansion (18) can be obtained within the frames of the so-called "uncoupled-multipole" approximation. This means that the rest terms in the series over m are truncated. The term a_0 takes account of these dropped out nonresonant multipoles.

The potential well U can be written down as a sum of two contributions: i) from the externally excited field U_{rf} and ii) from the beam-induced field U_b. For relatively short bunches only the first terms of Taylor expansion of U_{rf} can be retained:

$$U(\vartheta) \simeq \vartheta^2/2 + c_3\vartheta^3 + c_4\vartheta^4 + U_b(\vartheta) - \vartheta^2\frac{\partial U_b}{\partial(\vartheta^2)}(0). \qquad (19a)$$

Here, the rightmost term appears because, by the initial assumption, the incoherent synchrotron tune-shift is already embedded in the Ω_0^2-factor of Hamiltonian (1). The second-order quantities proportional to $c_{3,4}U_b$, U_b^2, are dropped out because of their small values. As far as the motion in a sinusoidal accelerating field is concerned, the nonlinearity coefficients $c_{3,4}$ acquire the form

$$c_3 = q\cot\varphi_s/6, \qquad c_4 = -\,q^2/24, \tag{19b}$$

where q is the RF harmonic number, φ_s is the stable phase angle ($\varphi_s > 0$ below transition, synchronous energy gain being proportional to $\cos\varphi_s$).

In the first-order approximation - for a negligibly small beam current - we take $U_b \equiv 0$. (The effect of the thus neglected stationary space charge self-fields is studied in Sect.4 of the paper.) Put $c_{3,4} = 0$ and $\Omega_s(\mathcal{E})/\Omega_0 = 1$ everywhere except for the resonant denominator where we take into account the natural nonlinearity of oscillations:

$$\left[\frac{\Omega_s(\mathcal{E})}{\Omega_0}\right]_{rf} \simeq 1 - \left[\frac{\Delta\Omega_s}{\Omega_0}\right]_{rf}\frac{\mathcal{E}}{\mathcal{E}_0}, \tag{20a}$$

$$\left[\frac{\Delta\Omega_s}{\Omega_0}\right]_{rf} \simeq \frac{3}{4}\,(5c_3^2 - 2c_4)\Delta\vartheta_b^2 << 1 \tag{20b}$$

where $\Delta\vartheta_b \simeq (2\mathcal{E}_0/\Omega_0^2)^{1/2}$ is the amplitude of oscillations along ϑ. The law of motion in linear approximation is $\vartheta(\mathcal{E},\psi) \simeq$ $\simeq (2\mathcal{E}/\Omega_0^2)^{1/2}\cos\psi$. Let us introduce both the reduced energy $\varepsilon = \mathcal{E}/\mathcal{E}_0$ and coordinate $x = \vartheta/\Delta\vartheta_b$. Let us make use of the new distribution function $f(\varepsilon) = F(\mathcal{E}_0\varepsilon)$ which is normalized to unity. From now on, we require that the derivative $f'(\varepsilon)$ vanishes at a sufficiently good pace both at the bunch center and its edge so as to provide the finite dimensions of the threshold map (i.e., the finite value of instability threshold).

Under such assumptions, solution (18) can be written down as:

$$\lambda_{m,r}(\Omega) \simeq -\,2\,\frac{\omega_s L}{M\,R}\,\frac{1}{\Delta\vartheta_b}\left\{\left[\frac{\Delta\Omega_s}{\Omega_0}\right]_{rf}^{-1}\mu_r^{(m)} + \delta\mu_r^{(m)}\right\},\quad Re\Omega \simeq m\Omega_s. \tag{21}$$

On the other hand, the stability criterion of Eq.(17) takes the form

$$h\,\mu_r^{(m)} < 1 \tag{22a}$$

for

$$|\mu_r^{(m)}| >> \left[\frac{\Delta\Omega_s}{\Omega_0}\right]_{rf}|\delta\mu_r^{(m)}|$$

where

$$h = -\,2\,\frac{\omega_s L}{M\,R}\,\frac{1}{\Delta\vartheta_b}\left[\frac{\Delta\Omega_s}{\Omega_0}\right]_{rf}^{-1}. \tag{22b}$$

The parameter h has a simple physical meaning. It coincides, up to the order of magnitude, with the ratio of coherent synchrotron tune-shift $\Omega - m\Omega_0$ (under the assumption of linear oscillations, $\Delta\Omega_s = 0$) to the natural spread of synchrotron frequencies inside the bunch. The dimensionless quantity $\mu_r^{(m)}$ is the r-th eigen-value of the normalized integral equation:

$$\mu^{(m)}(\varepsilon_*)j^{(m)}(x) = \int_{x^2}^{1}\frac{|f'(\varepsilon)|}{(\varepsilon-\varepsilon_*)}\,\frac{T_m(x/\sqrt{\varepsilon})}{\pi\sqrt{\varepsilon - x^2}}\,r_m(\varepsilon|j^{(m)})\,d\varepsilon, \tag{23a}$$

$$r_m(\varepsilon|j) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\cos m\psi\; j\left[\sqrt{\varepsilon}\cos\psi\right]d\psi,\quad \varepsilon_* = 0 \text{ or } 1 \tag{23b}$$

where $|x| \leqslant 1$, $T_m(x) = \cos(m\arccos(x))$ is a Chebyshev polynomial. The parameter ε_* determines the relative energy of resonant particles for which $\Omega_s(\varepsilon_*\mathcal{E}_0) = Re\Omega/m$. Therefore, the eigen-value $\mu(0) > 0$ corresponds to the resonant excitation of the bunch center, while $\mu(1) < 0$ - for that of the bunch edge.

The function $j_r^{(m)}$ can be interpreted as an approximate eigen-function of Eq.(12) reduced to the segment $[-1;1]$: $j_r^{(m)}(x) \sim J_l(\Delta\vartheta_b x)$, $l = (m,r)$, $Re\Omega \simeq m\Omega_s$. It can be seen that

$$j_r^{(m)}(\pm 1) = 0, \qquad \int_{-1}^{1} j_r^{(m)}(x)\,dx = 0,$$

$$j_r^{(m)}(-x) = (-1)^m\,j_r^{(m)}(x). \tag{24a}$$

From now on, we normalize $j_r^{(m)}(x)$ as follows:

$$\int_{-1}^{1}|j_r^{(m)}(x)|^2\,dx = 1. \tag{24b}$$

The eigen-functions of Eq.(23a) determine the set of the so called radial (with respect to the longitudinal phase-plane) modes of the m-th order multipole oscillations. They are mutually orthogonal throughout the cuts of the threshold map (i.e., for $\varepsilon_* \geqslant 1$, $\varepsilon_* \leqslant 0$). This becomes self-evident if, on changing the order of integration, one rewrites Eq.(23) in its equivalent form with a symmetric and real kernel:

$$\mu^{(m)}(\varepsilon_*)\, j^{(m)}(x) = \int_{-1}^{1} dx_1\, j^{(m)}(x_1) \int_{max(x^2,x_1^2)}^{1} \frac{|f'(\varepsilon)|}{(\varepsilon-\varepsilon_*)} \qquad (25)$$

$$\frac{T_m(x/\sqrt{\varepsilon})}{\pi\sqrt{\varepsilon - x^2}}\,\frac{T_m(x_1/\sqrt{\varepsilon})}{\pi\sqrt{\varepsilon - x_1^2}}\, d\varepsilon .$$

The Hermitian nature of the above equation allows one to evaluate the first-order additive correction $\delta\mu$ induced by multipole coupling (mixing). Having used the standard perturbation theory, one gets:

$$\delta\mu^{(m)}_r \simeq \sum_{m_1>0}\left[\int_0^1 |f'(\varepsilon)|\; r^2_{m_1}(\varepsilon|j^{(m)}_r)\; d\varepsilon\right] \times \qquad (26)$$

$$\begin{cases} 2m_1^2/(m^2 - m_1^2), & m_1 \neq m \\ -1/2 \quad , & m_1 = m . \end{cases}$$

The summation is performed over the multipoles $m_1 > 0$ of the same parity as m. In the first-order approximation, the multipoles of the opposite parity do not affect each other. This can be explained by their different symmetry properties (24a).

Physically, the term $\sim \delta\mu$ takes into account the effect of non-resonant excitation of multipoles. Such an excitation occurs due to deviation of phase-space perturbation pattern from the "single-wave" form $\sim e^{im\psi - i\Omega t}$ at frequencies $Re\Omega \simeq m\Omega_s$.

To solve eigen-value problem (23) by computer, we use the technique of the direct multiple iterations of the kernel. Such an approach is a quite justifiable one. Indeed, to estimate the threshold map cut dimensions one should look only for maximal-by-modulus eigen-values μ, λ, which are readily provided

by the above technique. The discretization of the problem is carried out by means of cubic-spline interpolation of the functions $j^{(m)}_1(x)$ and $r_m(\varepsilon{=}b^2|j)$ on an equidistant set of points in x,b-variables.

Let us give an example of an analytic solution which can also be used for the code-testing computations:

$$f(\varepsilon) = \frac{5}{2}(1-\varepsilon)^{3/2}, \quad \varepsilon_* = 1, \quad m = 1,$$

$$\mu^{(m)}_1(1) = -\frac{15}{8}, \quad \delta\mu^{(m)}_1(1) = -\frac{3}{16}, \quad j^{(m)}_1(x) = \begin{cases} \sqrt{3/2}\; x, & |x|<1 \\ 0, & |x|=1 \end{cases}.$$

The comparison of the above solution with the computed one is shown in Fig.2. The exact solution is drawn by a solid line, while the computational one - by a dashed line. The computations were carried out on the equidistant set of 11 points on segments

Testing of computer code

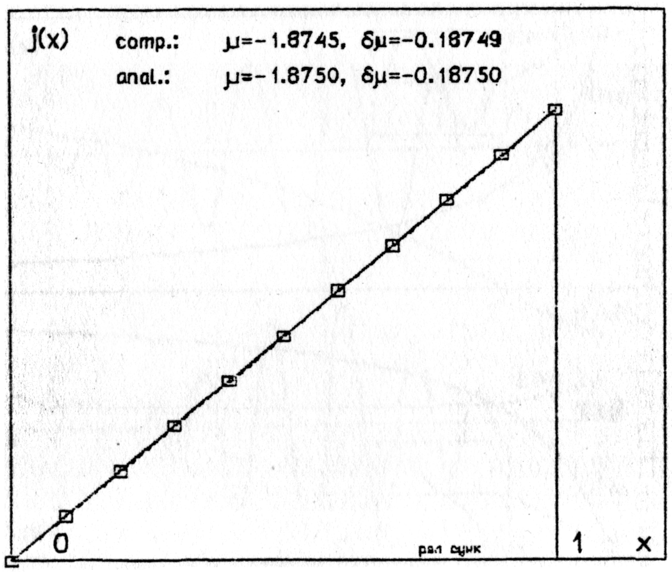

Fig.2. Computed and analytic solutions.

$0 \leqslant x,b \leqslant 1$. Spline nodes are marked by symbol □. The spline value at $x = 1$ is calculated via a continuity condition. Here, one should not make use of the first equality of Eqs.(24a) because the splined curve cannot provide a proper fitting of a discontinuous function. Under the condition of a more rapidly falling-off distributions $f(\varepsilon)$, the relevant eigen-functions would become continuous on the bunch edges.

For further computations, we take a more realistic distribution function $f(\varepsilon)$ made up of two conjugated parabolas:

$$f(\varepsilon) = \frac{3}{1-a^2}\times\begin{cases}(1-\varepsilon)^2 - a(1-\varepsilon/a)^2, & 0 \leqslant \varepsilon < a,\\ (1-\varepsilon)^2, & a \leqslant \varepsilon \leqslant 1, \quad 0 < a < 1.\end{cases} \tag{27}$$

Fig.3 shows the computed values of μ, $\delta\mu$ for $a = 0.5$ and $|m| = 1 \div 20$, $r = 1$. The quantities $\mu_1^{(m)}(0)$ and $\mu_1^{(m)}(1)$ vary approximately as $+3/|m|$, or $-4/|m|$ when $|m| > 10$. It should be

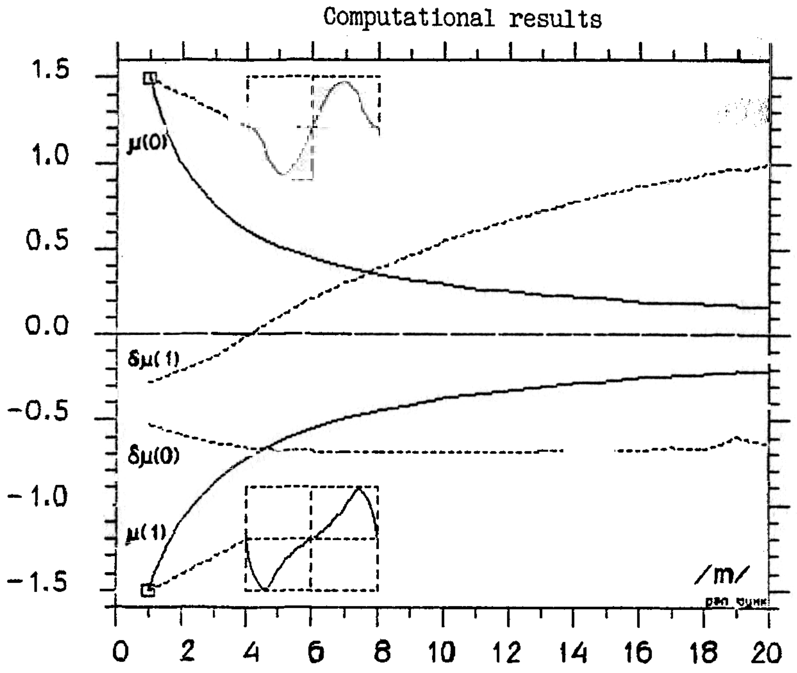

Fig.3. Eigen-values and correction terms.

reminded that $\mu(0) > 0$ corresponds to a resonant excitation of the bunch center, while $\mu(1) < 0$ – for that of its edge. The dipole oscillations appear to be the most dangerous, the relevant picture of line density perturbation being shown directly in Fig.3. But a further increase in beam intensity quickly results in a multipole mixing. Some higher radial modes of multipole oscillations can as well be excited. However, we do not show the plots of higher eigen-values. They would have run nearer to the abscissa axis. Due to these two effects, the observable picture of bunch line density perturbation quickly acquires a rather complicated pattern as the beam intensity grows. For $|\delta\mu_1^{(m)}| \sim 1$, $|\mu_1^{(m)}| \sim |m|^{-1}$, the obtained results remain valid up to $|m| \sim \Omega_0/\Delta\Omega_s$. This is a typical limitation on the applicability range of the "uncoupled-multipole" approximation, e.g. see[7].

Fig.4 shows a typical pattern of eigen-function $j_1^{(m)}(x)$ – bunch line density perturbation. As an example, we have taken the mode

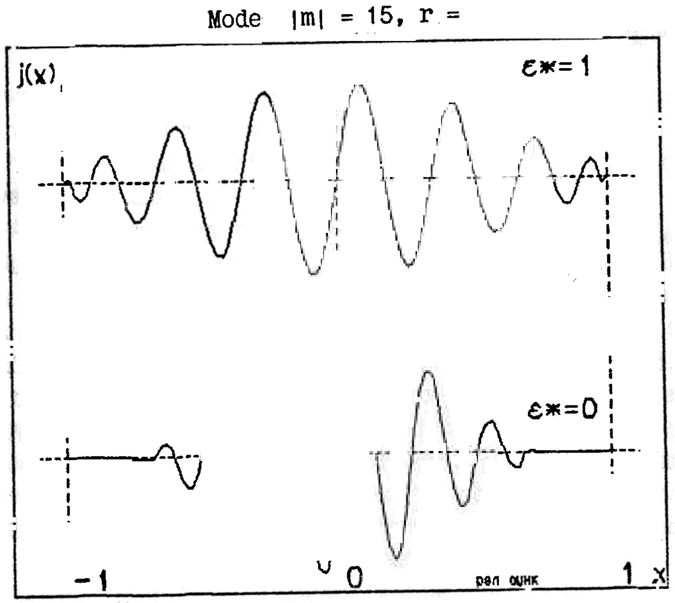

Fig.4. Bunch line density perturbation.

$|m| = 15$, $r = 1$ at the threshold of its excitation. As for other multipoles, the similar plot differs in the number of its zeros and, possibly, in the opposite symmetry of $j_1^{(m)}(x)$, see Eq.(24a). As it could be expected, for $\varepsilon_* = 0$ the perturbation tends to bunch center. In going from $\varepsilon_* = 0$ to $\varepsilon_* = 1$, the multipole content of phase-space perturbation varies as well. (Quantitatively, this content is characterized by functions (23b).) To the final extent, it explains the difference in the behavior of correction terms $\delta\mu_1^{(m)}(0)$ and $\delta\mu_1^{(m)}(1)$ observable in Fig.2.

For the practical purposes threshold criterion (22) can be replaced by a more severe inequality which ignores the sign of RL: $\max|\lambda_{m,r}| < 1$. On substituting here both Eq.(4) and Eq.(21), one gets an explicit form of limitation to be imposed on the value of $\omega_s L$:

$$\omega_s|L| = |Z_k(\Omega)/k| \leqslant$$

$$\leqslant \frac{\pi}{2\max|\mu_1^{(m)}|}\left[\frac{\Delta\Omega_s}{\Omega_0}\right]_{rf}\left\{\frac{\beta^2|\eta|(m_0c^2\gamma)}{e\,J_{ob}}\,B\left[\frac{\Delta p}{p_s}\right]^2\right. \qquad (28)$$

for

$$\left[\frac{\Delta\Omega_s}{\Omega_0}\right]_{rf}|\delta\mu_1^{(m)}| << |\mu_1^{(m)}|$$

where $J_{Ob} = J_0 q/M$ is the bunch current averaged along the RF period, $B = q\Delta\vartheta_b/\pi < 1$ is the bunch-factor. The first factor in the r.h.s. of Eq.(28) equals unity up to the order of its magnitude which follows from the computational results. The expression embedded in the braces coincides, though formally, with the r.h.s.e. of the well-known local criterion for stability of microwave oscillations[8]. The threshold of multipole oscillations is approximately $\Delta\Omega_s/\Omega_0$ times lower.

As an example, let us estimate the maximum tolerable value of the vacuum chamber low-frequency inductance for the 1-st Stage of the UNK. Consider a beam circulating at energy $m_0c^2\gamma = 600$ GeV, fixed-target operation regime, when $J_{Ob} = 1.4$ A; $|\eta| = 5\cdot10^{-4}$; bunch-factor $B = 0.38$; $\Delta p/p_s = 6.7\cdot10^{-4}$. On substituting these parameters into Eq.(28), one gets $\omega_s|L| < 3.3$ Ohm. (It will be shown in Sect.4 of the present paper that the effect of stationary

space charge self-fields somewhat slackens this limitation.)

To estimate the applicability range of the results obtained, we suppose that the impedance is of Eq.(9)-type, i.e. $L \simeq L_2$, for beam harmonics $|k| < k_*$. Usually, a typical frequency $k_*\omega_s$ is close to the cut-off frequency of the chamber.

Let us make use of the eigen-function computations (Fig.4). The frequency spectrum of the bunch line density perturbation for mode m, $r = 1$ is located near the harmonics of its high frequency modulation $k \sim \pm1.5(|m|+1)/\Delta\vartheta_b$. Up to the order of magnitude, this spectrum has half-bandwidth Δk equal to the inverse half-length of the modulation envelope, $\Delta k \sim s/\Delta\vartheta_b$ where $s \sim 3$. Therefore, $k_b \simeq \pm\left[1.5(|m|+1) \pm s\right]/\Delta\vartheta_b$. (In this context, it would be instructive to recall the well-known Radio-Engineering analogy - effect of the frequency translation of a video-pulse spectrum to a high frequency domain after multiplication of the video-pulse by a high frequency harmonic signal.) Thus, the results obtained are valid up to $|k_b| < k_*$ which is equivalent to

$$1.5\,\left[|m|+s'\right] \lesssim k_*\Delta\vartheta_b, \qquad s' \sim 3.$$

Here we neglect the higher radial modes. Practically, they exhibit sufficiently higher thresholds of excitation.

Let us continue with the example of the 1-st Stage of the UNK. Its vacuum chamber is $d = 7$ cm, revolution frequency is $\omega_s/2\pi = 14.4$ kHz and the ultimate harmonic is $k_* \simeq 2.3\cdot10^5$. Thus, $|m| < 10$.

4. THE EFFECT OF STATIONARY SPACE CHARGE SELF-FIELDS

From the formal viewpoint, this effect is described by terms of Eq.(19a) proportional to U_b, these terms being truncated in the above treatment. The theory of the question is fairly well developed[9]. As far as the problem under study is concerned, one can ignore the variation of the equilibrium bunch parameters $(\Delta\vartheta_b, \Delta p)$ caused by beam-induced field because

$$\frac{\delta\Delta\vartheta_b}{\Delta\vartheta_b} \sim -\frac{\delta\Delta p}{\Delta p} \qquad \frac{\omega_s L}{2MR}\,\frac{1}{\Delta\vartheta_b},$$

$$\left.\frac{2\omega_s L}{M\,R}\,\frac{1}{\Delta\vartheta_b}\right|_{threshold} \left[\frac{\Delta\Omega_s}{\Omega_0}\right]_{rf} \ll 1.$$

However, the relevant beam-induced modification of synchrotron nonlinearity exhibits itself as a more essential thing, its effect being determined by the ratio of the small parameters of the problem:

$$\frac{2\omega_s L}{M\,R}\,\frac{1}{\Delta\vartheta_b} \times \left.\left[\frac{\Delta\Omega_s}{\Omega_0}\right]^{-1}_{rf}\right|_{threshold} \qquad 1$$

Moreover, these corrections are introduced directly into the resonant denominator in the kernel of integral Eq.(23a) which enhances their effect drastically.

In the case of impedance (9), the beam-induced potential U_b has a simple form:

$$U_b(\vartheta) = \frac{2\pi\,J_{0b}\,\omega_s L}{q^2 V_{rf}\,\sin\varphi_s}\,\frac{N(\vartheta)-N(0)}{\int N(\vartheta)d\vartheta},\quad U_b(0) = 0 \tag{29}$$

where V_{rf} is the accelerating voltage amplitude per turn, $N(\vartheta)$ is the bunch line density. Up to the effects of the first order in U_b, one gets the following incoherent shift of the synchrotron frequency:

$$\left[\frac{\Omega_s(0)-\Omega_s(\mathcal{E})}{\Omega_0}\right]_b \simeq \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\Omega_0^3}{\Omega_s(\mathcal{E})\vartheta'(\mathcal{E},\psi)^2}\times \tag{30}$$

$$\times\left[\left[U_b(\vartheta(\mathcal{E},\psi)) - \vartheta^2(\mathcal{E},\psi)\frac{\partial U_b}{\partial(\vartheta^2)}(0)\right] - \ldots(\psi = 0)\right]d\psi$$

where $\vartheta(\mathcal{E},\psi{=}0) \equiv \max_\psi((\mathcal{E},\psi))$. The r.h.s. of the above Eq. contains the functions $\vartheta(\mathcal{E},\psi)$, $\vartheta'(\mathcal{E},\psi)$ and $\Omega_s(\mathcal{E})$ which are determined in a zero-order approximation. Naturally, one prefers linear oscillations to construct this initial approximation. Further on, we again replace variables $(\mathcal{E},\vartheta)$ by a pair of the reduced ones

(ε,x). We also employ the line density function $n(x) = N(\Delta\vartheta_b x)$ which is reduced to segment $[-1;1]$ and normalized to unity. In such notations, Eqs.(29), (30) can be combined to yield:

$$\left[\frac{\Omega_s(0)-\Omega_s(\varepsilon)}{\Omega_0}\right]_b \simeq -2\,\frac{\omega_s L}{M\,R}\,\frac{1}{\Delta\vartheta_b}\times \tag{31}$$

$$\left[\frac{1}{2}\frac{\partial n}{\partial(x^2)}(0) - \frac{2}{\pi}\int_0^{\pi/2}\sin^2\varphi\,\frac{\partial n}{\partial(x^2)}\left[\sqrt{\varepsilon}\sin\varphi\right]d\varphi\right].$$

We shall continue to use the distribution function of Eq.(27). One can easily find out that the relevant line density $n(x)$ can be written as:

$$n(x) = \frac{16}{5\pi}\,\frac{1}{1-a^2}\times\begin{cases}(1-x^2)^{5/2} - a^{3/2}(1-x^2/a)^{5/2}, & 0 \leqslant x^2 < a\\ (1-x^2)^{5/2}, & a \leqslant x^2 \leqslant 1.\end{cases} \tag{32}$$

The contribution from the natural nonlinearity is given by Eq.(20a). Therefore, on substituting $n(x)$ from Eq.(32) into Eq.(31), one immediately arrives at the net nonlinearity law:

$$\left[\frac{\Omega_s(0)-\Omega_s(\varepsilon)}{\Omega_0}\right]_{rf+b} = \left[\frac{\Delta\Omega_s}{\Omega_0}\right]_{rf}\times\left[\varepsilon + h\,G(\varepsilon,a)\right], \tag{33a}$$

$$G(\varepsilon,a) = \frac{1}{1-a^2}\left|\,g(\varepsilon) - \sqrt{a}\,g(\varepsilon/a) - \frac{4}{\pi}(1-\sqrt{a})\right] \tag{33b}$$

The parameter h is defined quantitatively by Eq.(22b). But here this parameter acquires a somewhat different qualitative interpretation. This time, up to a factor of the order of (-2), it is the ratio of small-amplitude incoherent synchrotron tune-shift $\left[\Omega_0(J_0{\neq}0) - \Omega_0(J_0{=}0)\right]$ to the natural inside-the-bunch spread of synchrotron frequencies. (Thus, as is well known, for the impedance of Eq.(9) both incoherent and coherent synchrotron tune-shifts practically coincide by their absolute values while having the opposite signs.) Function $g(\varepsilon)$ can be reduced to a quadrature via elliptic integrals. Nevertheless, computationally it is more convenient to retain its explicit integral representation:

$$g(\varepsilon) = (16/\pi^2)\int_0^{\varphi'} \sin^2\varphi \left[1-\varepsilon \sin^2\varphi\right]^{3/2} d\varphi, \qquad (33c)$$

$$\varphi' = \begin{cases} \pi/2, & \varepsilon \leqslant 1 \\ \arcsin(1/\sqrt{\varepsilon}), & \varepsilon > 1. \end{cases}$$

Now, one can easily modify Eq.(22a) to account for the leading stationary self-field effect. As a result, the stability criterion becomes a nonlinear function of variable h:

$$\frac{h\,\mu_r^{(m)}(\varepsilon_*|h)}{1 + h\,G(1,a)} < 1 \qquad (34a)$$

for

$$\frac{|h\,\mu_r^{(m)}(\varepsilon_*|h)|}{|1 + h\,G(1,a)|} \gg \left[\frac{\Delta\Omega_s}{\Omega_0}\right]_{rf} |h\,\delta\mu_r^{(m)}(\varepsilon_*|h)|. \qquad (34b)$$

The quantity $\mu_r^{(m)}(\varepsilon_*|h)$ is an eigen-value of an integral equation which differs from Eq.(23a) in the expression for its resonant denominator: $(\varepsilon - \varepsilon_*)\to(w(\varepsilon) - \varepsilon_*)$, where $w(\varepsilon)$ can be referred to as a normalized nonlinearity law of synchrotron oscillations:

$$w(\varepsilon) = \frac{\varepsilon + h\,G(\varepsilon,a)}{1 + h\,G(1,a)}, \quad w(0) = 0, \quad w(1) = 1. \qquad (35)$$

The estimates show that up to the near-threshold values $|h| \lesssim 1$ the $w(\varepsilon)$-function varies monotonously within the range $0 \leqslant w(\varepsilon) \leqslant 1$. Thus, the solution for the relevant eigen-value problem can be easily obtained by computer via the same above mentioned technique.

Fig.5 shows the plot of the r.h.s. of Eq.(34a) versus parameter h for $|m| = 1$, $a = 0.5$. Two tangent lines present the analogous dependence resulting from the first-order approximation, i.e. from Eq.(22a). The dashed lines show the plots of the "$h\delta\mu_1^{(1)}(\varepsilon_*|h)$"- function which allow an easy verification of inequality (34b). It can be seen that the effect of stationary space charge self-fields widens the region of stable parameters by a factor of 2.0 for $h < 0$, or by 1.4 for $h > 0$.

This change in the threshold is caused by a stationary distortion of distribution of particles over the incoherent

synchrotron frequencies. It can be explained by a competition of the following two factors.

Mode $|m| = 1$, $r = 1$

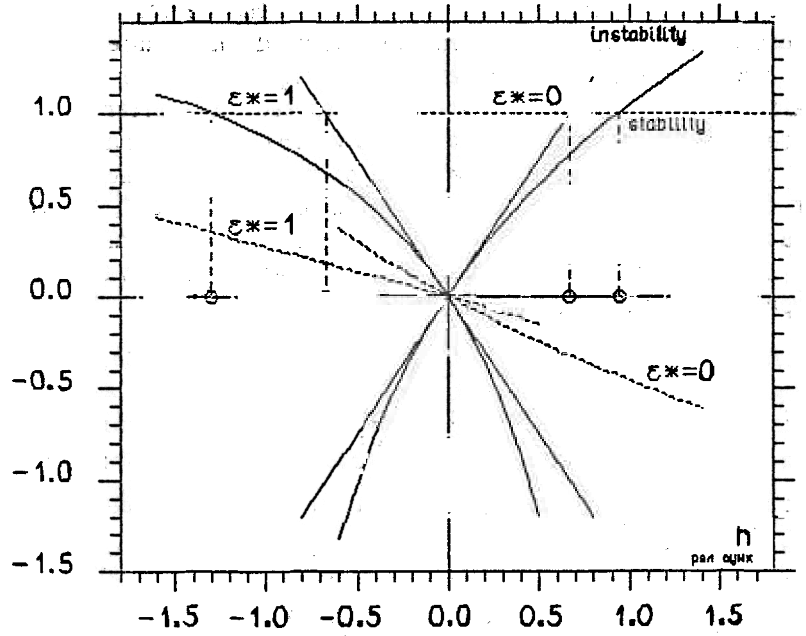

Fig.5. Effect of stationary self-fields.

On the one hand, the total spread $(\Delta\Omega_s/\Omega_0)_{rf+b}$ decreases for $h > 0$ (destabilizing effect), and, otherwise, increases for $h < 0$ (stabilization).

On the other hand, the relative number of particles with their synchrotron frequencies lying close to the resonant value decreases irrespective of the sign of h. Formally, this effect manifests itself in

$$\frac{\mu_r^{(m)}(\varepsilon_*{=}0|h{>}0)}{\mu_r^{(m)}(\varepsilon_*{=}0|h{=}0)} < 1, \qquad \frac{\mu_r^{(m)}(\varepsilon_*{=}1|h{<}0)}{\mu_r^{(m)}(\varepsilon_*{=}1|h{=}0)} < 1.$$

Such a behavior of the eigen-values is caused by the decreasing value of the resonant factor $(w(\varepsilon) - \varepsilon_*)^{-1}$ as compared to the natural nonlinearity when $w(\varepsilon) = \varepsilon$. Indeed, for all ε from the range $0 \leqslant \varepsilon \leqslant 1$

$$w(\varepsilon) \geqslant \varepsilon \quad \text{when } h > 0,\ \varepsilon_* = 0,$$
$$w(\varepsilon) \leqslant \varepsilon \quad \text{when } h < 0,\ \varepsilon_* = 1,$$

which can be confirmed computationally. The second of the above factors always tends to widen the region of the stable parameters. It has a more significant effect as compared to the beam-induced variation of the total synchrotron spread. A similar picture is observed for other distributions $(\alpha \neq 0.5)$, see Appendix.

Therefore, the acceptable value of the low-frequency inductance of the vacuum chamber for the 1-st Stage of the UNK, contrary to that cited after Eq.(28), can be raised as high as $\omega_s|L| < 4.7$ Ohm. For the 2-nd Stage this value varies approximately proportionally to $\gamma^{-5/4}$.

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APPENDIX

Variation of the free parameter α of the functions in Eqs.(27), (32) and (33) allows one to simulate a rather wide collection of realistic distributions. However, in doing this, as well as in attempting to analyze the higher multipoles, one encounters some technical problems. These are caused by non-monotonous behavior of functions $\Omega_s(\mathcal{E}) = \Omega_{s\ rf+b}(\mathcal{E})$ and $w(\varepsilon)$ at the near-threshold values of h.

Formally, the effect of the non-monotonous nonlinearity $\Omega_s(\mathcal{E})$ can be coped with by treating Ω_s as a new independent variable, $\mathcal{E} \to \Omega_s(\mathcal{E})$, and by introducing $F'(\Omega_s) = \sum \left[F'\ |d\Omega_s/d\mathcal{E}|^{-1}\right](\mathcal{E}(\Omega_s))$ as a new distribution function in the interval $(\Omega_{s\ min} \div \Omega_{s\ max})$. Here, the summation is performed over the regions of $\Omega_s(\mathcal{E})$- monotonicity where the single-valued function $\mathcal{E} = \mathcal{E}(\Omega_s)$ does exist.

Analysis of Eqs.(14) shows that now the generalized threshold curve can run to infinity, $|Re\lambda_t|, |Im\lambda_t| \to \infty$. It would take place in the points of resonant $(\Omega = m\Omega_s(\mathcal{E}))$ excitation of the particles which are moving along inner $(F'(\Omega_s) \neq 0)$ phase-plane trajectories with locally extreme values of synchrotron frequencies $(\Omega_s'(\mathcal{E}) = 0)$.

Nevertheless, as far as the purely imaginary inductive impedance is concerned, this "blow-up" of the threshold diagram occurs in its inessential part and should not be interpreted as a loss of stability of the system. Suffice it to mention the following consideration.

For $h > 0$ the finite value of instability threshold (if there is any) corresponds to resonant excitation of particles at phase-plane trajectory with $\Omega_s = \Omega_{s\ max}$ on the necessary condition of $F'(\Omega_{s\ max}) = 0$. (Respectively, for $h < 0$ – on the trajectory with $\Omega_s = \Omega_{s\ min}$ when $F'(\Omega_{s\ min}) = 0$.) However, calculations by computer of Eqs.(33) and (35) show that for a wide range of the h-parameter values (up to the threshold of octupole oscillations) these requirements are still met on the resonant phase-plane trajectories of the zero-order (in h-parameter) approximation. Namely, either at $\varepsilon_* = 0$ for $h > 0$, or at $\varepsilon_* = 1$ for $h < 0$.

Therefore, the account of stationary self-field effects for different values of α and lower multipoles, $|m| \leq 4$, can well be considered without any modifications to the computational technique. Naturally, in the process of these computations the net nonlinearity law $\Omega_{s\ rf+b}(\mathcal{E})$ should be controlled. Results for dipole and quadrupole oscillations are listed in Tables 1,2.

Table . Threshold values of parameter h $(|m| = 1)$.

Account of stationary effects	$h < 0,\ \varepsilon_* = 1$ yes	$h < 0,\ \varepsilon_* = 1$ no	$h > 0,\ \varepsilon_* = 0$ no	$h > 0,\ \varepsilon_* = 0$ yes
$a = 0.1$	−0.92	−0.72	0.19	0.28
0.2	−1.23	−0.72	0.31	0.44
0.3	−1.35	−0.71	0.42	0.59
0.4	−1.36	−0.69	0.54	0.76
0.5	−1.30	−0.67	0.67	0.94
0.6	−1.19	−0.63	0.81	1.13
0.7	−1.04	−0.57	0.95	1.33
0.8	−0.85	−0.50	1.11	1.55
0.9	−0.60	−0.39	1.27	1.78

Table 2. Threshold values of parameter h $(|m| = 2)$.

Account of stationary effects	$h < 0,\ \varepsilon_* = 1$ yes	$h < 0,\ \varepsilon_* = 1$ no	$h > 0,\ \varepsilon_* = 0$ no	$h > 0,\ \varepsilon_* = 0$ yes
$a = 0.1$	−1.45	−1.07	0.30	0.52
0.2	−2.47	−1.05	0.47	0.78
0.3	−3.00	−1.02	0.64	1.04
0.4	−3.08	−0.97	0.82	1.31
0.5	−2.82	−0.92	1.01	1.60
0.6	−2.40	−0.85	1.21	1.91
0.7	−1.92	−0.76	1.43	2.25
0.8	−1.41	−0.65	1.65	2.61
0.9	−0.91	−0.49	1.90	3.00

В.И.Балбеков, С.В.Иванов.

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